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Math 1060 □□ Project 2
Graphing with MAPLE

Introduction

The purpose of this project is to investigate how trigonometric functions are transformed using the same procedures as the functions you studied in MATH 1050. The math-graphing program, MAPLE V, is available, with instructions, on the computers in the Math Lab.

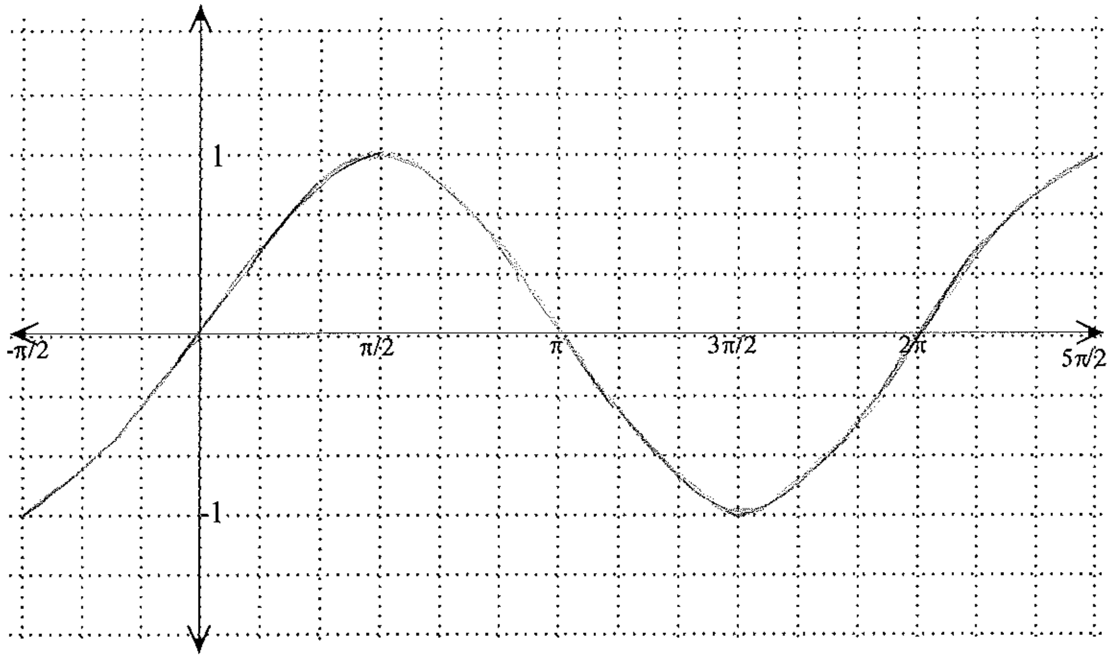
Getting started:

1. Turn on the computer you're using.
2. Type username: **math**
   Leave the password cell blank.
   Press the Enter Key.
3. Double-click on the MAPLE icon.
4. You **do not** need to **save** you work to a disk **when you close**, or quit, your MAPLE work for this assignment.

Part I: BASIC SINE AND COSINE GRAPHS

1. Graph y = sin x by typing exactly: **plot (sin (x) , x = -.5*Pi..2.5*Pi , y = -1.5..1.5);**
   Press the Enter Key.
   This graphs the function from $\frac{-\pi}{2} \le x \le \frac{5\pi}{2}$.

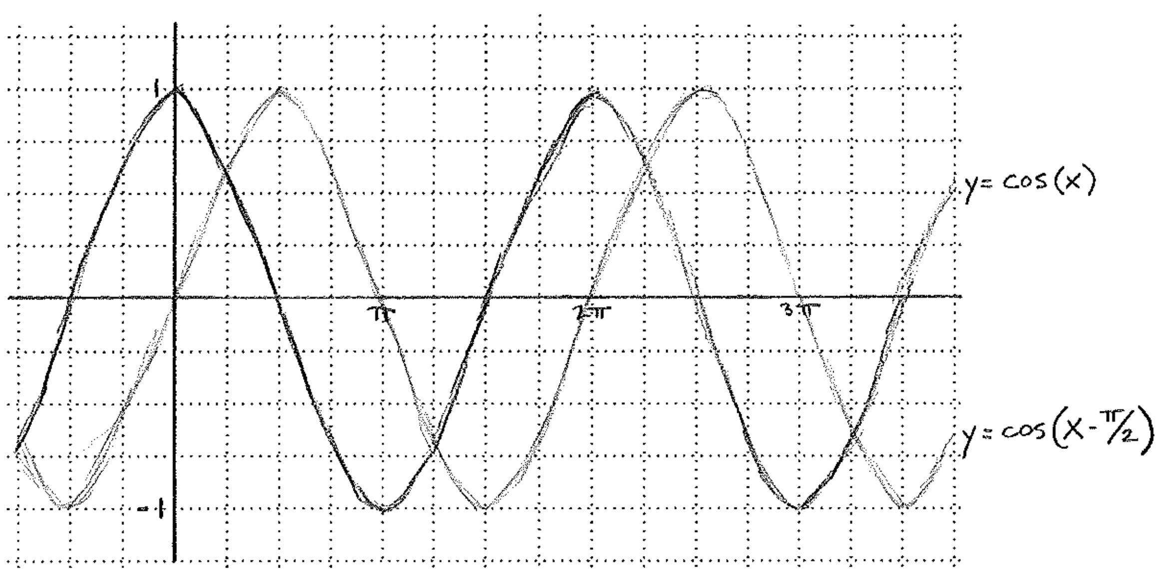
Do not print your graph. Copy it to the grid below, making sure to follow the π-units on the x-axis, and the whole number units on the y-axis. How many periods does your graph show? THE GRAPH SHOWS 1½ PERIODS.



2. Graph $y = \cos x$ and $y = \cos\left(x - \frac{\pi}{2}\right)$ on the same graph by typing exactly:

plot ({ cos (x), cos (x – Pi / 2) }, x = -.5*Pi..2.5*Pi);

Copy your graph onto the grid below making sure to mark the x-axis with π-units and the y-axis with whole number units. Label each function: MAPLE uses different colors for the different functions, but you should write the names for each of them.



Explain how the graph of a function changes when $\frac{\pi}{2}$ is subtracted from the variable x in the argument. THE GRAPH WILL PHASE SHIFT $\pi/2$ ALONG THE X AXIS IN THE POSITIVE DIRECTION.

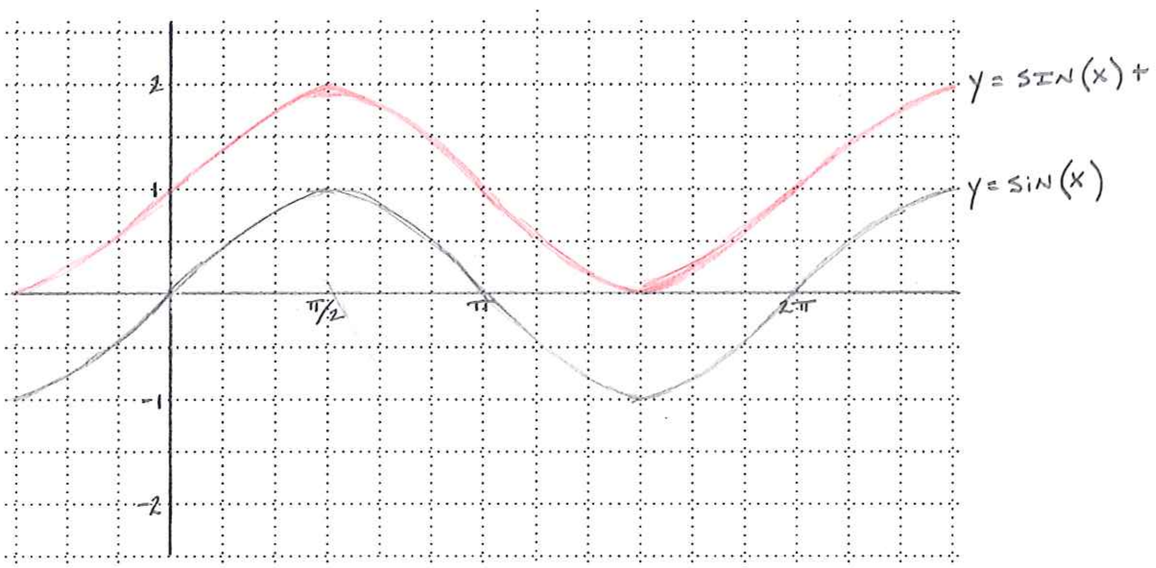
3. Use MAPLE to observe the graphs of $y = \cos x$ and $y = \cos\left(x + \frac{\pi}{2}\right)$. How does the graph of a function change when $\frac{\pi}{2}$ is added to the variable x in the argument?

THE GRAPH WILL PHASE SHIFT $\pi/2$ ALONG THE X AXIS IN THE NEGATIVE DIRECTION.

4. Which of the two previous transformations of $y = \cos x$ looks the same as $y = \sin x$?

THE $y = \cos(x + \pi/2)$ MORE CLOSELY RESEMBLES $y = \sin(x)$.

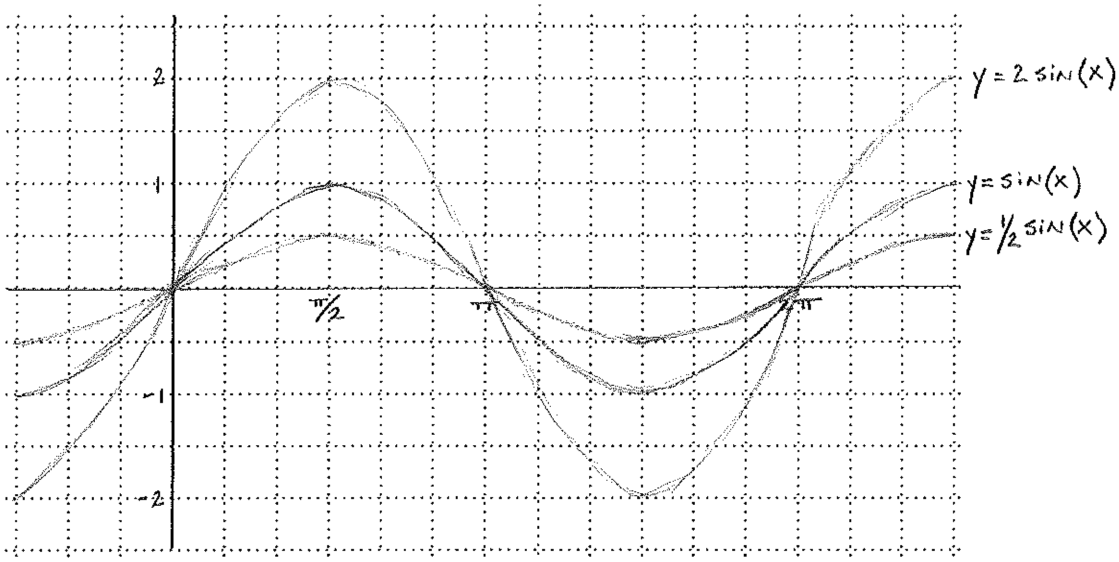
5. Use MAPLE to graph $y = \sin x$ and $y = \sin(x) + 1$ on the grid below. Mark the x-axis with π-units, and make it larger than the period of either function. Mark the y-axis with enough whole number units to show the total range. Label each function.



Explain how the graph of a function changes when a number is added to it.

THE GRAPH WILL SHIFT UP OR DOWN ALONG THE Y AXIS ACCORDINGLY.

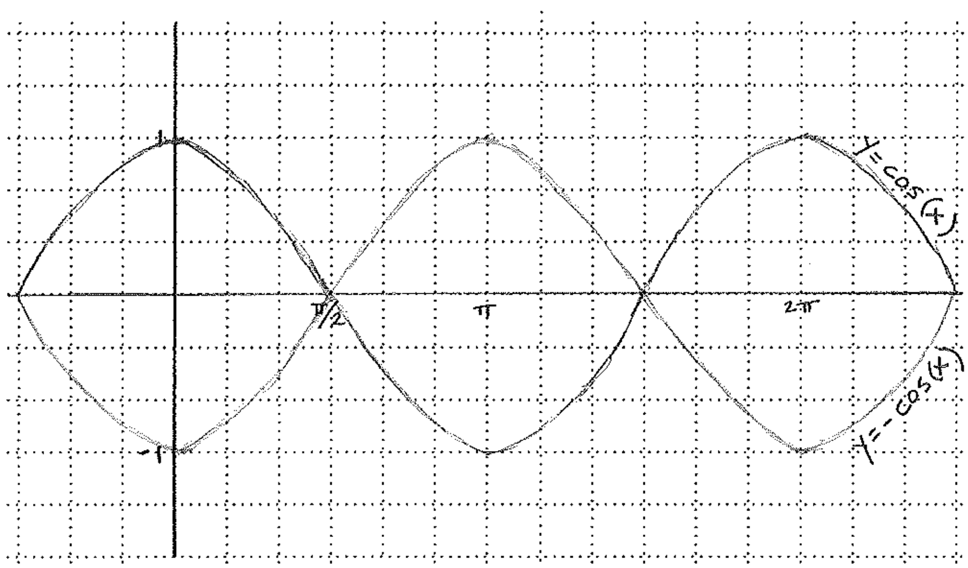
6. Use MAPLE to graph $y = \sin x$, $y = 2\sin x$, and $y = \frac{1}{2}\sin x$ on the same grid below. Be sure to mark the x-axis with enough π-units to show a complete period of each function, and mark the y-axis with whole number units. Label each function.



Explain how the graph of a function changes when a number is multiplied or divided by it. THE AMPLITUDE IS INCREASED OR DECREASED ACCORDINGLY.

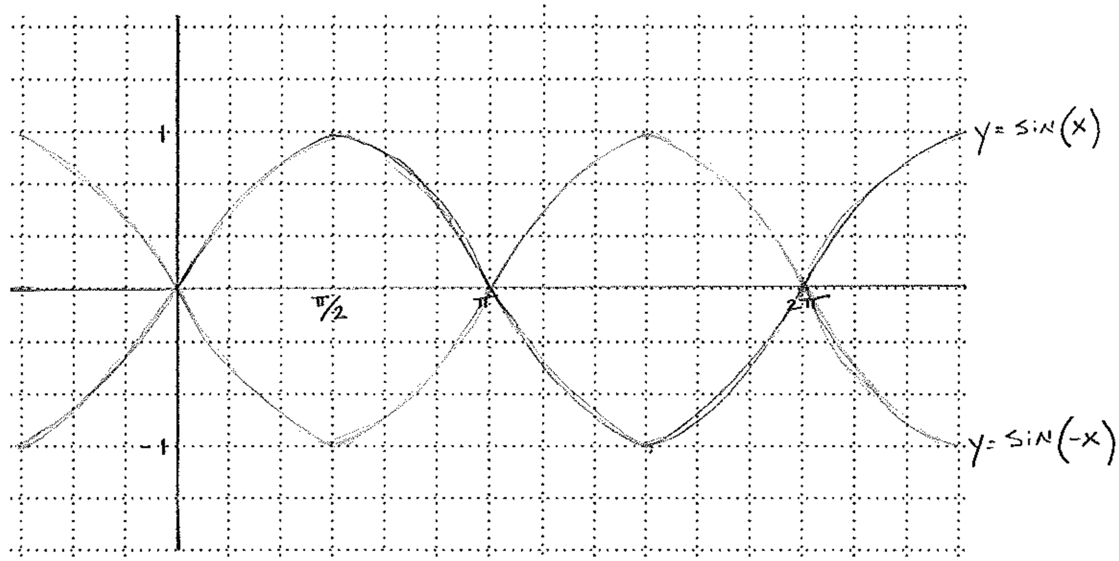
Part II MORE SINE AND COSINE GRAPHS

1. Use MAPLE to graph $y = \cos x$ and $y = -\cos x$ on the grid below. Label each function and explain how a graph changes when a function is multiplied by a negative.



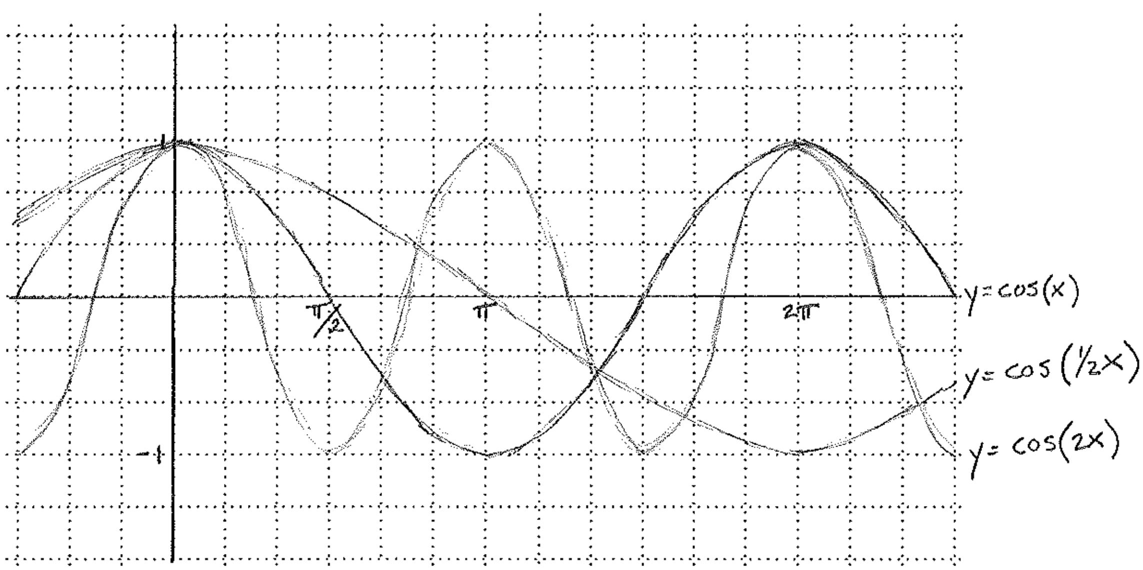
A GRAPH MULTIPLIED BY A NEGATIVE WILL FLIP THE GRAPH ABOUT THE X AXIS TO CREATE AN INVERSE OF THE ORIGINAL GRAPH.

2. Use MAPLE to graph y = sin x and y = sin (-x) on the same graph. Label each function and explain how a graph changes when the variable is multiplied by a negative.



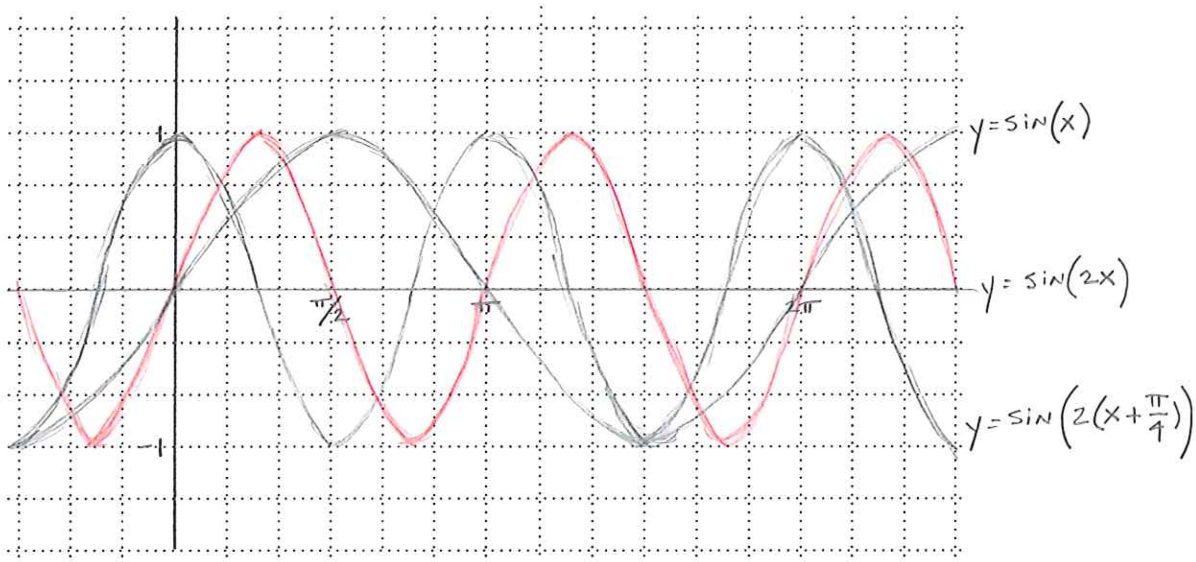
A GRAPH MULTIPLIED BY A NEGATIVE WILL FLIP THE GRAPH ABOUT THE X AXIS TO CREATE AN INVERSE OF THE ORIGINAL GRAPH.

3. Use MAPLE to graph $y = \cos x$, $y = \cos(2x)$, and $y = \cos(\frac{1}{2}x)$ on the grid below. Mark the x-axis with enough π-units to show the largest period of the three functions. Label each function and explain how multiplying or dividing a variable by a whole number changes the graph of a trigonometric function.



A GRAPH WILL SHORTEN ITS PERIOD WHEN MULTIPLIED BY A WHOLE NUMBER AND WILL LENGTHEN ITS PERIOD WHED MULTIPLIED BY A FRACTION LESS THAN ONE.

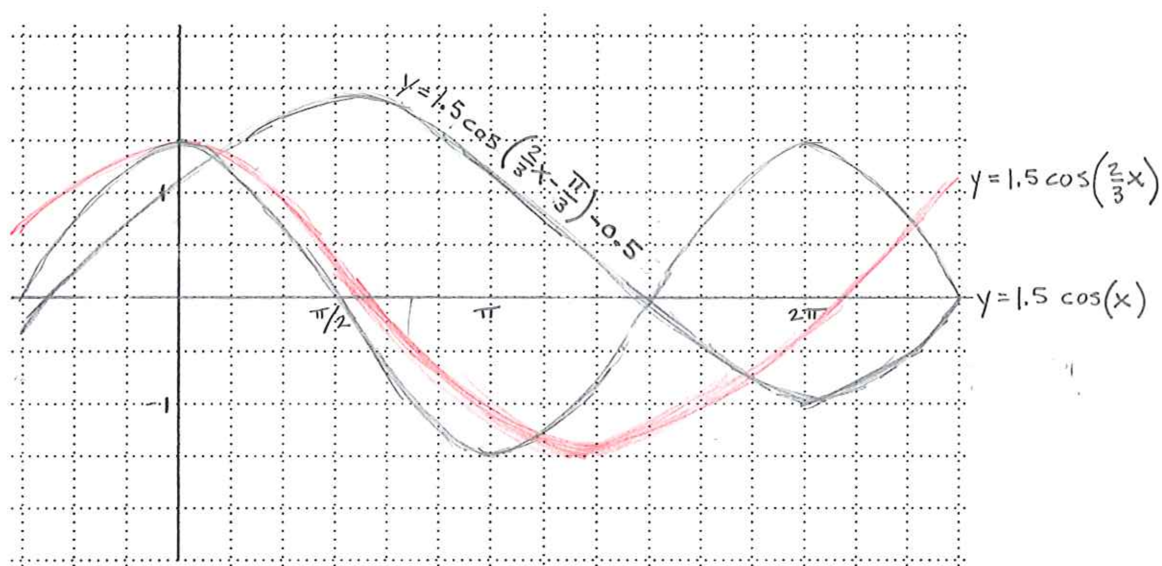
4. Use MAPLE to graph $y = \sin x$, $y = \sin(2x)$, and $y = \sin(2x + \frac{\pi}{2})$ on the grid below. Explain **how** the period and the shift are effected when the variable is multiplied by two.



THE PERIOD WILL BE SHORTENED WHEN MULTIPLIED BY TWO. THE PHASE SHIFT IN THIS CASE WILL BE HALVED FROM $\frac{\pi}{2}$ to $\frac{\pi}{4}$ AND WILL SHIFT $\frac{\pi}{4}$ TO THE LEFT

5. Use MAPLE to graph $y = 1.5\cos x$, $y = 1.5\cos\left(\frac{2x}{3}\right)$, and $y = 1.5\cos\left(\frac{2x}{3} - \frac{\pi}{3}\right)$ - 0.5.

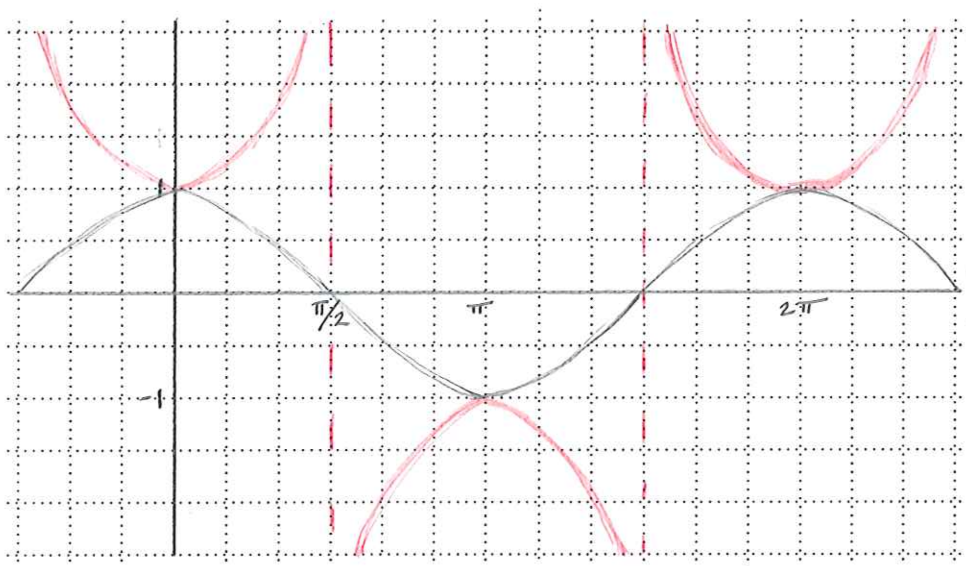
Explain **how** the period and the shift are effected when the variable is multiplied by two-thirds.



THE PERIOD WILL BE LENGTHENED WHEN MULTIPLIED BY 2/3. THE PHASE SHIFT WILL CHANGE TO $\frac{\pi}{2}$ AND WILL SHIFT ALONG THE X AXIS TO THE RIGHT.

Part III OTHER TRIGONOMETRIC FUNCTION GRAPHS

1. Use MAPLE to plot y = cos x and y = sec x on the grid below by typing exactly:
   plot ({ cos (x), sec (x)}, x = -Pi..2*Pi, y = -3..3);
   What do the vertical lines represent? They should not be part of your graph.



THE RED DASHED LINES ARE THE VERTICAL ASYMTOTES. THEY REPRESENT THE LIMITS THAT THE GRAPH OF y = sec(x) CANNOT CROSS.

Use MAPLE to find the following pairs of values by typing exactly:

eval (cos (*x*), x = -Pi/2); and then **eval (sec (*x*), x = -Pi/2);**

The message "Error, singularity encountered," means that no value exists, and in this case, the graph suggests $\pm\infty$; however, "does not exist" is the best answer.

$\cos\frac{-\pi}{2} = $ <u>O</u>, $\sec\frac{-\pi}{2} = $ <u>ER</u> $\qquad$ $\cos\frac{-\pi}{3} = $ <u>.5</u>, $\sec\frac{-\pi}{3} = $ <u>2</u>

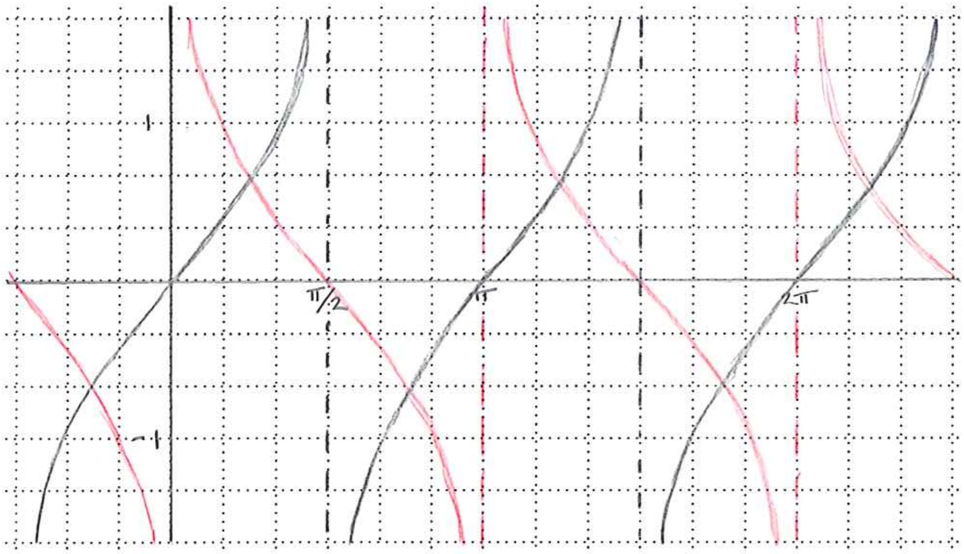
$\cos 0 = $ <u>1</u>, $\sec 0 = $ <u>1</u> $\qquad$ $\cos\frac{2\pi}{3} = $ <u>-.5</u>, $\sec\frac{2\pi}{3} = $ <u>-2</u>

$\cos\frac{3\pi}{4} = $ <u>-.71</u>, $\sec\frac{3\pi}{4} = $ <u>-1.41</u> $\qquad$ $\cos 3.0 = $ <u>-.99</u>, $\sec 3.0 = $ <u>-1.01</u>

What relationship between the cosine and secant do the graph and these values suggest?

THE AMPLITUDE OF A COSINE GRAPH WILL ALSO REPRESENT THE MAX AND MIN OF THE SECANT GRAPH.

2. Use MAPLE to plot y = tan x and y = cot x on the grid below by typing exactly:
   plot ({ tan (x), cot (x)}, x = -Pi..2*Pi, y = -3..3);
   Use MAPLE to find the following pairs of values:



tan 0 = O ,   cot 0 = ER

$\tan\frac{\pi}{2}$ = ER , $\cot\frac{\pi}{2}$ = ER

$\tan\frac{-2\pi}{3}$ = 1.73 , $\cot\frac{-2\pi}{3}$ = .58

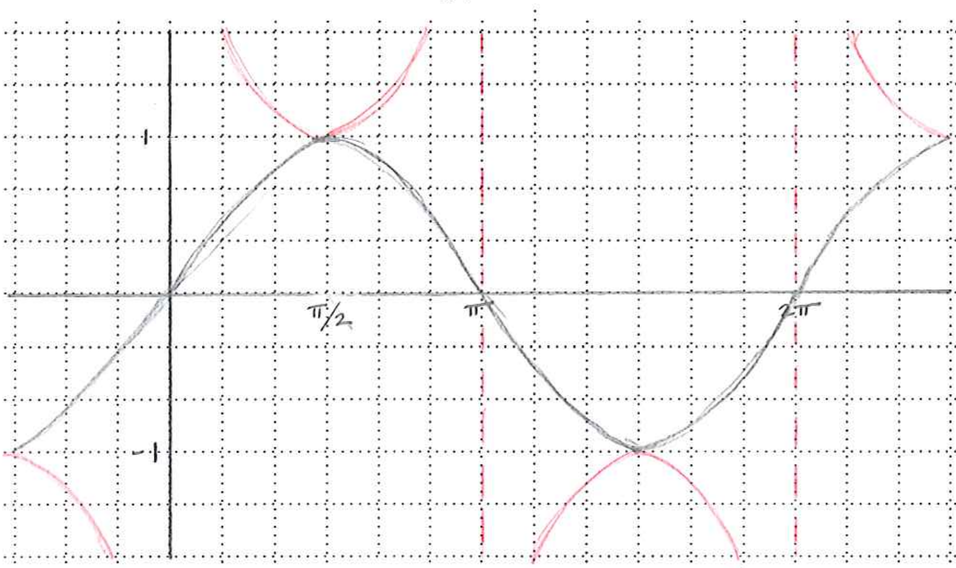
tan –2.159 = 1.49 , cot –2.159 = .67

$\tan\frac{-\pi}{4}$ = -1 , $\cot\frac{-\pi}{4}$ = -1

$\tan\frac{\pi}{3}$ = 1.73 , $\cot\frac{\pi}{3}$ = .58

What relationship between the tangent and cotangent do the graph and paired values suggest? THE POINT WHERE y=tan(x) CROSSES THE X AXIS IS THE ASYMTOTE OF y=cot(x) AND VISA VERSA.

3. Use MAPLE to plot y = sin x and y = csc x on the grid below by typing exactly:
   plot ({ sin (x), csc (x)}, x = -Pi..2*Pi, y = -3..3);
   Use MAPLE to find the following pairs of values:



$\sin 0 =$ 0 , $\csc 0 =$ ER $\sin \frac{\pi}{2} =$ 1 , $\csc \frac{\pi}{2} =$ 1

$\sin \frac{-\pi}{3} =$ -.87 , $\csc \frac{-\pi}{3} =$ -1.15 $\sin \frac{\pi}{6} =$ .5 , $\csc \frac{\pi}{6} =$ 2

$\sin \frac{-3\pi}{4} =$ -.71 , $\csc \frac{-3\pi}{4} =$ -1.41 $\sin 3.35 =$ -.21 , $\csc 3.35 =$ -4.8

What relationship between the sine and cosecant do the graph and paired values suggest?

THE AMPLITUDE OF $y = \sin(x)$ REPRESENTS THE MAX AND MIN VALUES OF $y = \csc(x)$

Part IV COMBINING TRIG FUNCTIONS

Use MAPLE to graph $y = 0.6 \sin(2x)$, $y = \cos(x - \frac{\pi}{3})$, and $y = 0.6 \sin(2x) + \cos(x - \frac{\pi}{3})$ on the same grid by typing exactly:

plot ({.6*sin(2*x), cos(x-Pi/3), .6*sin(2*x)+cos(x-Pi/3)}, x=-Pi/2..2.5*Pi,y=-1.6..1.6);

Fill in the chart and explain how the third function appears to relate to the first two functions.

	$\frac{-\pi}{2}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$
$y = 0.6 \sin(2x)$	0	0	0.6	0	-0.6	0
$y = \cos(x - \frac{\pi}{3})$	-1	.5	1	.5	0	-.5
$y = 0.6 \sin(2x) + \cos(x - \frac{\pi}{3})$	-1	.5	1	0	-.5	-.5

How are the columns in row 3: $y = 0.6 \sin(2x) + \cos(x - \frac{\pi}{3})$

related to the columns in row 1: $y = 0.6 \sin(2x)$ and row 2: $y = \cos(x - \frac{\pi}{3})$?

UNKNOWN

Make changes to the period, phase shift, and amplitude in the last combined function. "Copy" the combined function: $y = 0.6 \sin(2x) + \cos(x - \frac{\pi}{3})$ and paste it in other "plot" commands: plot (0.6 sin (**1.5**x) + cos (x - $\frac{\pi}{3}$),x=-Pi/2..2.5*Pi,y=-1.6..1.6); and

plot (0.6 sin ($2x$) + cos (**1.5**x - $\frac{\pi}{3}$) x=-Pi/2..2.5*Pi,y=-1.6..1.6); and plot (0.6 sin (**1.5**x) + cos (**1.5**x - $\frac{\pi}{3}$), x=-Pi/2..2.5*Pi,y=-1.6..1.6); and

plot (0.6 sin (2x) + cos (**1.5**x + $\frac{\pi}{3}$), x=-Pi/2..2.5*Pi,y=-1.6..1.6); and

plot (0.6 sin (2x) + cos (x + $\frac{\pi}{3}$), x=-Pi/2..2.5*Pi,y=-1.6..1.6); and

plot (**1.2** sin (2x) + cos (x - $\frac{\pi}{3}$), x=-Pi/2..2.5*Pi,y=-1.6..1.6); and

plot (0.6 sin (2x) + cos (x - $\frac{\pi}{2}$), x=-Pi/2..2.5*Pi,y=-1.6..1.6); and

and so on.

Plot each of your changes using MAPLE and see how they affect the graph.

After you have graphed at least twenty variations of the problem, write one or two paragraphs explaining how you observed the graphs change with the changes in the constants of the function. NO COMMENT AT THIS TIME.