SLCC – Fall 2010

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Math 1060 □ Project 1
Classical Trigonometry

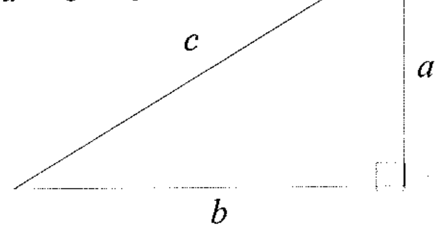
Introduction

The purpose of this project is to appreciate some roots of trigonometry.

Part I: PYTHAGOREAN FORMULA

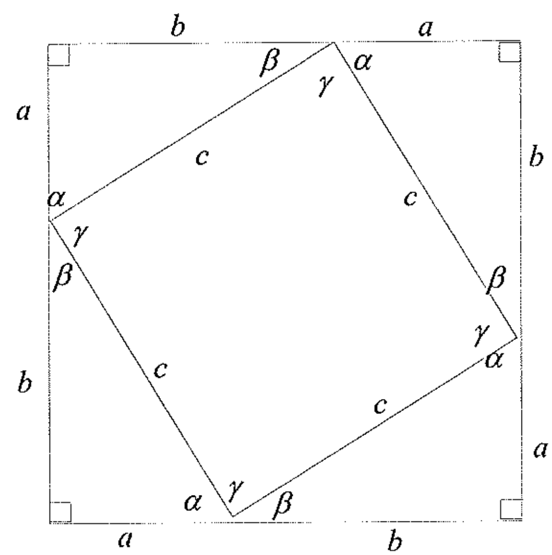
One of the most fundamental formulas in mathematics, and for trigonometry in particular, is the Pythagorean formula named for a Greek mathematician and philosopher who live from about 580 to 496 BC. It states that the sum of the squares of the lengths of the two legs of a right triangle is equal to the square of the length of the hypotenuse of that triangle.

$a^2 + b^2 = c^2$



This famous relationship between the sides of a right triangle was known and used by the Babylonians 1500 years before Pythagoras proved it. The Egyptians learned it from the Babylonians, and by the time of Pythagoras, it was so well known that the Chinese also knew the relationship.

Now there are hundreds of proofs for this famous theorem – and thousands of false proofs.
The Indian mathematician Bhaskara gave the following geometric proof about 1150.



Fill in the missing parts.

We begin with the information that the large quadrilateral is a square.

1. The length of each side of the large square is $a + b$
2. The area of the large square is $(a + b)^2$
3. The four right triangles are congruent because side a, angle between, and side b are equal
4. The four hypotenuses, side c, are congruent because Question #3 proved the triangles congruent
5. The four angles α are congruent, and the four angles β are congruent because the triangles are congruent by SAS therefore all angles are congruent
6. The measure of $\alpha + \beta =$ 90° because of Right Triangle properties
7. The measure of angle $\gamma =$ 90° because $\alpha + \beta + \gamma = 180°$
8. The inscribed figure is a square because All four sides are equal and all interior angels measure to be 90°
9. The area of the inscribed square is c^2
10. Write an alternative expression for statement 2, the area of the large square, by combining the small square with the area of the four right triangles: $\frac{1}{2}(a \cdot b) \cdot 4 + c^2$
11. Write the expression for the large square in statement 2 equal to the expression for the large square in statement 10, then simplify the equation.

$$(a+b)^2 = \frac{1}{2}(a \cdot b) \cdot 4 + c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$\left\{ a^2 + b^2 = c^2 \right\}$$

If a and b are the two legs of a right triangle with hypotenuse c, as described above, use the Pythagorean theorem to find the missing side:

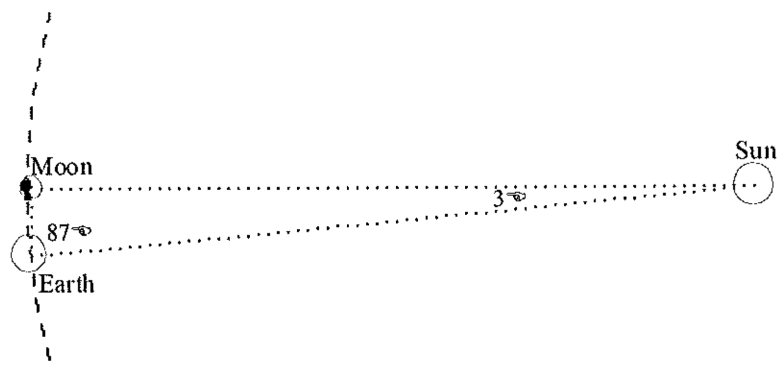
12. $a = 12$ cm, $b = 5$ cm, $c =$ 13 cm
13. $a = 20$ cm, $b =$ 21 cm, $c = 29$ cm
14. $a =$ 12 in, $b = 35$ in., $c = 37$ in.
15. $a = 130$ m, $b = 144$ m, $c =$ 194 m
16. $a = \frac{6}{25}$ in., $b =$ 7/100 in, $c = \frac{1}{4}$ in.
17. $a =$ $4\sqrt{5}$ ft, $b = 8$ ft., $c = 12$ ft.
18. $a = 3\sqrt{5}$ units, $b = 3$ units, $c =$ $3\sqrt{6}$ u
19. $a =$ N/A, $b = 0.8$ m, $c = 0.66$ m
20. $a = 6.25$ cm, $b =$ 6.5625 cm, $c = 9.0625$ cm
21. $a =$ $5\sqrt{7}$ mi, $b = 15$ mi., $c = 20$ mi.

Part II: ANCIENT ASTRONOMY

About 150 A.D., Ptolemy, a Greek astronomer, formalized the popular belief that the Earth was the center of the Universe. That theory prevailed in Europe and the Middle East for more than 1300 years until Copernicus successfully argued that the Earth, and the planets, revolved around the Sun. However, about 350 years before Ptolemy, another Greek astronomer and mathematician reasoned that the Earth circled the Sun and he calculated the relative distance between the Earth and the Moon, and between the Earth and the Sun.

Aristarchus (310 – 230 BC) was born on island of Rhodes and studied with a student of Aristotle in the Egyptian city of Alexandria when he was in his twenties. He reasoned that the Moon shines by reflecting the Sun's light, and that when exactly half of the Moon shines, the angle formed from the Moon to the Sun, and from the Moon to the Earth is a right angle. Then he measured the angle between the Moon and the Sun, and found the ratio of those distances.

$$\frac{dist(Earth, Moon)}{dist(Earth, Sun)}$$



Use your calculator's sine or cosine function to determine this ratio as a decimal number.

$\sin(3°) = .052366$

Approximate the ratio as a fraction $\frac{1}{'whole' number}$, the way ancient mathematicians (and bookies in Las Vegas) would write it.

$1/.052366 = \{19.1\}$

Use this fraction to tell how many times farther the Sun is from the Earth than the Moon is, according to Aristarchus' measurements.

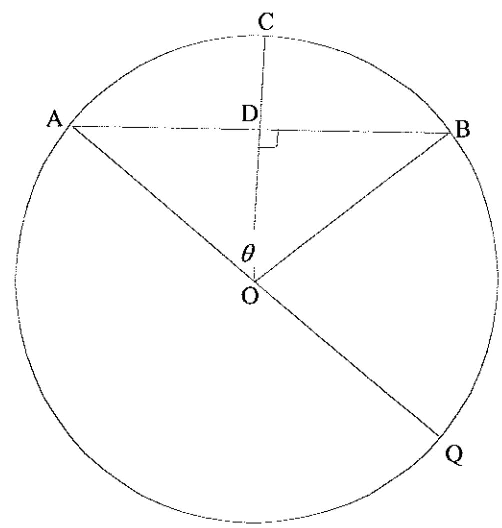
19 TIMES AS FAR

Although Aristarchus' logic was good, the ratio he calculated was wrong because he could not tell exactly when it was a half moon, and his measuring instruments were primitive. The correct ratio is closer to $\frac{1}{200}$. What angle would he have measured between the Moon and the Sun to get this ratio? (You will either do a lot of trial and error angles to find this ratio, or you will have to work the inverse of a trig function.)

$\sin^{-1}\left(\frac{1}{200}\right) = \{.28648°\}$

Part III: TRIGONOMETRIC RATIOS

Of course the ancient Greeks didn't have calculators to help with their calculations. They didn't even have the trigonometric ratios we use today. Instead of sine, tangent, and so on, early mathematicians used the length of a chord, such as AB, which subtends a central angle θ. They compared the chord to the diameter AQ of a circle which they divided into 120 parts: crd $(\theta) = \frac{AB}{AQ} = \frac{AB}{120}$



The Greeks made a table of chords for every ⅓ of a degree in a circle from 0° to 180°.

Give the following values:

crd (0°) = 0

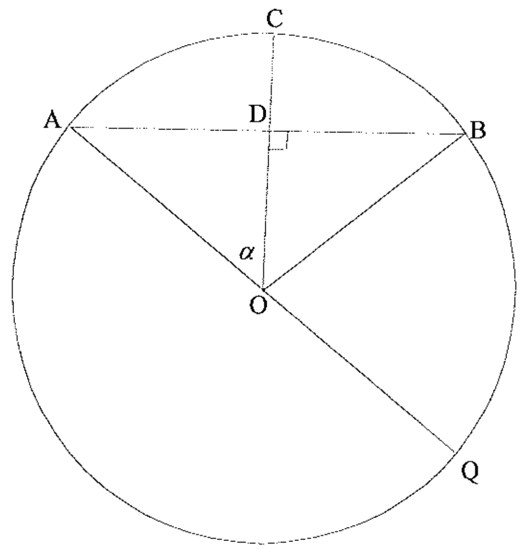
crd (180°) = $\frac{120}{120} = 1$

crd (90°) = 1/2

crd (60°) = 1/3

crd (120°) = 2/3

crd (30°) = 1/6



We worked the celestial ratio problem above using a right triangle and the trigonometric functions that we are familiar with, rather than using chords and circles as Aristarchus did. For more than a thousand years mathematicians used the comparison of chords to the diameter of a circle to do trigonometry. In the late ninth century AD, Al-Battani, a Persian astronomer and mathematician, organized trigonometric ratios for the sides of right triangles, and we still use his method. He made tables of half chords. If α is an angle which is half of angle θ, and crd (θ) uses the chord AB, then the sine of α, which is half of θ, uses half of chord AB = AD. Which, if either, of the following is true? Justify your answer by either showing steps or examples to explain how you decided whether #1is true, #2 is true, or neither is true.

1. crd θ = sin α

$\mathrm{crd}(\theta) = \frac{\theta}{180}$

$\alpha = \frac{1}{2}\theta$

$2\alpha = \theta$

$\sin(\alpha) = \frac{2\alpha}{180}$

$\sin(\alpha) = \frac{\alpha}{90}$

2. ½ crd θ = sin α

$\frac{1}{2}\mathrm{crd}(\theta) = \sin(\alpha)$

$\mathrm{crd}(\theta) = 2\sin(\alpha)$

$\frac{\theta}{180} = 2\sin(\alpha)$

$\sin(30) = \frac{1}{2}$

$\frac{\frac{\theta}{180}}{2} = \frac{2\sin(\alpha)}{2}$

$\sin(\alpha) \neq \frac{\theta}{360}$